***2023 PURE MATHEMATICS MOCK PAPERS***

**WAKATA**

SECTION A (40 MARKS)  
Answer all questions in this section.

1. Solve the equation: . (05 marks)
2. Given that and .  
   Show that (05 marks)
3. Find if . (05 marks)
4. A straight line joining the points and is parallel to the line joining points and . Find the values of and . (05 marks)
5. and are two points whose coordinates are respectively and is a point . Show that . (05 marks)
6. Evaluate (05 marks)
7. The quadratic equation has equal roots. Find the roots of the equation. (05 marks)
8. Solve the differential equation; (05 marks)

SECTION B (60 MARKS)  
Answer any five questions from this section. All questions carry equal marks.

1. (a) The polynomial is exactly divisible by and leaves a remainder when divided by . Find the values of and . (07 marks)  
   (b) Given that and are factors of . Show that . (05 marks)
2. (a) Given that . Find . (06 marks)  
   (b) The volume of a liquid in a container is given by ; where is the depth of the liquid. The liquid is leaking from the container. It is observed that, when the depth of the liquid is , the depth is decreasing at a rate of per hour. Find the rate at which the volume of liquid in the container is decreasing at the instant when the depth is . (06 marks)
3. (a) The lines and are perpendicular. Find the value of . (04 marks)  
   (b) Find the coordinates of the point where the line through and crosses the plane . (08 marks)
4. Expand in ascending powers of , as far as the term in . (12 marks)
5. (a) Sketch the curve . (06 marks)  
   (b) The area bounded by the curve and the line is rotated about the -axis through one revolution. Determine the volume of the solid generated. (06 marks)
6. (a) Prove that: (08 marks)  
   (b) Solve the equation for . (04 marks)
7. (a) Find the equation of the ellipse whose focus is , the directrix and accentricity equal to . (04 marks)  
   (b) Points and are a part and it is determined from the sound of an explosion heard at those points at different times that the location of the explosion is closer to than . Show that the location of the explosion is restricted to a hyperbola whose equation is . (08 marks)
8. In a culture, the bacteria count is 100,000. The number is increased by in 2 hours. In how many hours will the count reach 200,000, if the rate of growth of bacteria is proportional to the number present? (12 marks)

JJEB

SECTION A  
Answer all the questions in this section

1. A geometric progression has the sum of the first and second terms equal to . If the sum of the fourth and the fifth terms is 108. Calculate the  
   (i) first term,  
   (ii) Common ratio of the progression (5 marks)
2. Show that is an ellipse and hence determine its center and eccentricity. (5 marks)
3. Find the coordinates on the curve for which the tangent is a horizontal line. (5 marks)
4. Solve the equation for . (5 marks)
5. The line and are given by the equation and respectively.  
   Find the,  
   (i) value of for which and intersect.  
   (ii) point of intersection. (5 marks)
6. Prove by induction that for all positive integers . (5 marks)
7. Evaluate . (5 marks)
8. In order to post a parcel, the sum of the circumference of a cylindrical parcel and its height should add up to . Find the dimensions of the largest parcel that can be accepted. (5 marks)

SECTION B (60 MARKS)  
Answer any five questions in this section. All questions carry equal marks.

1. (a) Given that and   
   Determine the following in the form :  
   (i)   
   (ii) . (7 marks)  
   (b) Find the locus of the complex number represented by . (5 marks)
2. (a) If is a tangent to a circle .  
   prove that .  
   Hence find the equation of the tangents from origin to the circle (7 marks)  
   (b) Find the equation of a circle whose centre is at and touches the line joining and .

(5 marks)

1. Given the curve .  
   Determine the;  
   (a) range of values for in which the curve does not lie and hence find the coordinates of the turning point.  
    (6 marks)  
   (b) asymptotes and sketch the curve   
    (6 marks)
2. (a) Given the line . Find the :  
   (i) value of if the line is in the plane   
   (ii) distance of the point from the line. (7 marks)  
   (b) Given points and find the coordinates of point which divides externally in the ratio 3:8. (5 marks)
3. (a) Prove that .  
   Hence solve the simultaneous equations:

(7 marks)  
(b) Find the first three terms of the Binomial expansion of . (5 marks)

1. (a) Show that . Hence if , Prove that .  
    (6 marks)  
   (b) Given that A, B and C are angles of a triangle, prove that;
2. (a) Differentiate the following with respect to :  
   (i) .  
   (ii) (6 marks)  
   (c) Given that , Prove that . (6 marks)
3. (a) Solve the differential equation . (3 marks)  
   (b) In an agricultural plantation the proportion of the total area that has been destroyed by a bacterial disease is . The rate of the destruction of the plantation is proportional to the product of the proportion already destroyed and that not yet. It was initially noticed that half of the plantation had been destroyed by the disease and that at this rate another quarter of the plantation would be destroyed in the next 6 hours.  
   (i) Form a differential equation relating and time   
   (ii) Calculate the percentage of the population destroyed 12 hours after the disease was noticed.

(9 marks)

UMTA

SECTION A

1. Solve for given , solve for in the equation .  
    (05 marks)
2. Prove that and are parametric equation of a parabola. Find its vertex, focus and length of latus rectum.  
    (05 marks)
3. Given and are angles of a triangles prove that . (05 marks)
4. Differentiate from first principles (05 marks)
5. Find the square root of . (05 marks)
6. (05 marks)
7. Using small changes find the to 4 significant figures. (05 marks)
8. Find the angle between the planes and . (05 marks)  
   **SECTION B**
9. (a) When a polynomial is divided by the remainder is 3 and when divided by the remainder is 1 . Prove that when divided by the remainder will be . (06 marks)  
    (b) Find the term independent of in the expansion of . (06 marks)
10. (a) Find the region where the curve does not lie, hence the determine the turning points and their nature. (04 marks)  
    (b) State the asymptotes and intercept. (03 marks)  
    (c) Sketch the curve. (05 marks)
11. (a) A man pays premium of 100 dollars at the beginning of every year to an insurance company on an understanding that at the end of the 15 years they can receive back the premium he had paid with compound interest. What did he receive? (06 marks)

(b) A committee of six is to be formed from nine women and three men in how many ways can this be chosen so as to include at least one man.  
 (06 marks)

1. Partialize

(12 marks)

13 (a) Solve the differential equation .  
 (04 marks)  
(b) The rate at which malaria spreads in the body is proportion to the number of infected cells in the body. If the number of infected cells in the body at any time is . Given that after 1 month the number of cells infected is doubled and considering the initial number of cells infected to be   
(i) Show that (ii) Show that five months later the number of the infected cells is . (08marks)  
14. (a) Prove that .  
 (06 marks)  
(b) Solve for from to . Given that   
 (06 marks)  
15. (a) Given . Show that   
 (04 marks)  
(b) Differentiate and simplify (08 marks)  
16. (a) Find the vector equation of the line of intersection between the planes

(b) Using the dot product, find the equation of the plane containing points and   
 (06 marks)

**UTEC**

**SECTION A**Answer ALL questions in this section

1. Use the Echelon method to solve the simultaneous equations:

(05 marks)

1. Prove the identify: (05 marks)
2. . Calculate the total area bounded by the curve , the -axis and the lines and .  
    (05 marks)
3. Find a unit vector perpendicular to the vectors;

(05 marks)

1. A circle whose centre lies in the first quadrant touches the positive axis at +4 , and touches the line . Find the radius of the circle, and state the coordinates of its centre. (05 marks)
2. Given that and are real numbers such that: , where , find the modulus of .  
    (05 marks)
3. Differentiate the function from first principles.  
    (05 marks)
4. A curve is represented by the parametric equations; , find the equation of the tangent to the curve at the point .  
    (05 marks)

**SECTION B (60 MARKS)**

1. Given the lines and .  
   (a) Find the coordinates of their point of intersection. (04 marks)  
   (b) Calculate the acute angle between the lines. (04 marks)  
   (c) Find the Cartesian equation of the plane containing the lines (04 marks)
2. (a) The roots of the equation differ by 3 , find the possible values of .  
   (05 marks)  
   (b) Use the remainder theorem to find the remainder when the polynomial is divided by .  
    (07. marks)
3. . (a) Given that , show that ; hence or otherwise solve for in the interval . (06 marks)  
   (b) Given that and are angles of a triangle. Prove that; .

(06 marks)

1. (a) Use small changes to evaluate to .  
    (05 marks)  
   (b) Evaluate: to .  
    (07 marks)
2. (a) The term of a series is . Calculate the sum of the first 20 terms of the series.  
    (05 marks)  
   (b) Expand up to the term in . State the range of values of within which the expansion is convergent. Hence evaluate; to .  
    (07 marks)

A diagram of a triangle with lines and triangles

Description automatically generated

is right pyramid with a square base. The pyramid is completely inscribed in a sphere of radius , where is the centre of the sphere. is the centre of the square base BCDE as shown. Given that .  
(a) Show that the volume of the pyramid;

(07 marks)  
(b) Calculate the maximum volume of the pyramid.

1. (a) Show that the equation of the chord joining the point and on the parabola is

(04 marks)

(b) If the chord in (a) above passes through the point , hence:

(i) show that the chord makes a right angle at the origin .  
(ii) find the locus of the mid-point of . (08 marks)

1. In a certain game reserve, there are 80 elephants. Poachers start killing the elephants at a rate which is directly proportional to the number of elephants remaining in the forest. After one month 40 elephants have been killed. Let be the number of elephants killed after months.  
   (a) Show that;   
   (b) Calculate the: (07 marks)  
   (i) number of elephants killed after 2 months.  
   (ii) time taken to kill 75 elephants, and in this case state the average number of elephants killed per day.  
     
    (05 marks)

END

WAKISHA

SECTION A (40 MARKS)  
Answer all questions in this section.

1. If , show that (05 marks)
2. Given that . Hence prove that (05 marks)
3. Evaluate (05 marks)
4. Solve equation for . (05 marks)
5. Show that the circles whose equations are and cut orthogonally. ` (05 marks)
6. Expand as far as the term in . By putting in your expansion, estimate , correct to two decimal places. (05 marks)
7. Find the perpendicular distance of a point from the line .  
    (05 marks)
8. An inverted cone with vertical angle has water in it dripping out through a hole at the vertex at the rate of per minute. Find the rate at which it's level will be decreasing at an instant when the volume of water left in the cone is . (05 marks)

**SEECTION B**

1. (a) Given that; . Find;  
   (i) modulus of . (2 marks)  
   (ii) argument of . (2 marks)  
   Hence express in polar form. (2 marks)  
     
   (b) Show the region represented by on an argand diagram and state the locus complex number represented by the wanted region.  
    (06marks)
2. (a) and are points whose position vectors are and respectively. Determine the position vectors of a point that divides line internally in the ratio 5:1.  
    marks)  
   (b) If vectors and are parallel to a plane containing point . Determine;  
   (i) the equation of the plane.  
    (04 marks)  
   (ii) the angle the line makes with the plane in (i) above.  
    (04 marks)
3. The curve is given parametrically by the equations and   
   (a) Find the Cartesian equation of the curve.  
    (02 marks)  
   (b) Determine the turning points of the curve.  
   (05 marks)  
   (c) Sketch the curve.  
    (05 marks)
4. (a) If , Prove that   
    (06 marks)  
   (b) By using a suitable substitution ,  
   evaluate   
    (06 marks)
5. (a) Prove that .  
    (05 marks)  
   (b) Show that . Hence solve the equation correct to 4 significant figures.  
    (07 marks)
6. If the line is a tangent to an ellipse .

prove that .  
 (04 marks)  
Hence determine;  
(i) equations of four common tangent to the ellipses

(04 marks)  
(ii) the equations of the tangents at the point to ellipse

(04 marks)  
15. (a) The eighth term of an arithmetic progression is twice the third term and the sum of the first eight terms is 39. Find the first three terms of the progression and show that its sum to term is .  
 (06marks)  
(b) Find how many terms of the series . must be taken so that the sum will differ from the sum to infinity by less than .  
 (06 marks)  
16. On March 2020. There were 60 female antelopes kept a side to feed lions. It was discovered that the rate at which the antelopes were eaten was proportional to sum of 5 and the number of antelopes present at any given time per month. On 31st August. 40 antelopes were present.  
(a) Form a differential equation and solve it. (09 marks)  
(b) How many antelopes were left by end of November, 2020?  
(Assume each month is 30 days and none of antelope dies on itself on.)  
 (03 marks)

**END**

***Add more if you have please and share. Thanks in advance***